Reading Time: An initial **2 minutes** to view **BOTH** sections

MATHEMATICS METHODS : UNITS 3 & 4, 2021

Test 2 – (10%)

3.2.1 to 3.2.22 (not 3.2.5), 3.1.1 – 3.1.6, 3.1.9

Time Allowed	First Name	Surname	Marks
25 minutes			25 marks

Circle your Teacher's Name:

Mrs Alvaro	Mrs Bestall	Ms Chua
Mr Gibbon	Mrs Greenaway	Mr Luzuk
Mrs Murray	Ms Robinson	Mr Tanday

Assessment Conditions: ***(N.B. Sufficient working out must be shown to gain full marks)***

- Calculators: Not Allowed
- Formula Sheet: Provided
- Notes: Not Allowed

PART A – CALCULATOR FREE

Question 1 **[2, 2, 2, 2 — 8 marks]**

Differentiate the following, **do not simplify your answer**:

a) $\frac{d}{dx}\left(e^{x^2} + \pi\cos x\right)$

$= 2xe^{x^2} - \pi\sin x$ ✓ ✓

b) $\frac{d}{dx}\, xe^{\sin x}$

$= x\cos x\, e^{\sin x} + e^{\sin x}$ ✓ ✓

c) $\frac{d}{dx}\sin(4x^2 - 3)$

$= \cos(4x^2-3)\ 8x$ ✓ ✓

d) $\int_x^0 (3t^2-1)^3 dt \; - \; 3x^2$

$= -(3x^2-1)^3 \; - \; 6x$ ✓ ✓

must have -ve.

Question 2

Determine the following: **[2, 2, 2 — 6 marks]**

a) $\int (x^2+5)^2 dx$

$= \int (x^2+5)(x^2+5)\, dx$ ✓expands

$= \int x^4 + 10x^2 + 25\, dx$ correctly.

$= \dfrac{x^5}{5} + \dfrac{10x^3}{3} + 25x + c.$

✓ corret antidiff allow follow though.

b) $\int 2x^4 e^{x^5-3} dx$

$= \frac{2}{5} \int 5x^4 e^{x^5-3}\, dx$ ✓

$= \frac{2}{5} e^{x^5-3} + c.$

✓correct adjustment

(-1 for parts a&b if no c at least once)

c) $F'(3)$ given $F(x) = \int_1^x \frac{1}{1+t^2} dt.$

$F'(x) = \dfrac{d}{dx} \int_1^x \dfrac{1}{1+t^2}\, dt$

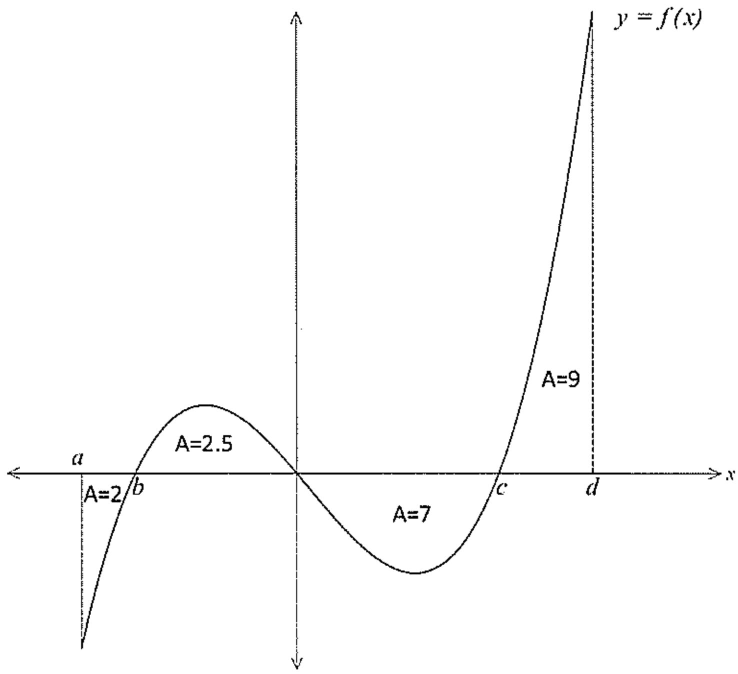
$= \dfrac{1}{1+x^2}$ ✓

$F'(3) = \dfrac{1}{10}$ ✓ (answer only o.k)

Question 3 **[1, 1, 1, 2, 1 — 6 marks]**

The graph of $y=f(x)$ is shown below and the area of the various regions are as indicated.

Determine



a) $\int_0^c 3f(x)dx$

$= -21$ ✓

b) $\int_0^a f(x)dx$

$= -0.5$ ✓

c) The area enclosed between the graph of $y=f(x)$ and the x-axis from $x=b$ to $x=d$.

$2.5+7+9 = 18.5$ units^2 ✓

(ignore units^2)

d) The value of $\int_0^d (x-f(x))dx$ $= \int_0^d x\,dx - \int_0^d f(x)\,dx$

$= \left[\frac{x^2}{2}\right]_0^d - (-7+9)$

$= \frac{d^2}{2} - 2$ ✓ Value $\int_0^d f(x)\,dx$ correct.

✓ R|W

e) What value of m gives $\int_0^m f(x)\,dx$ the greatest value for $x=a$ to $x=d$.

d ✓

Question 4 **[1, 4— 5 marks]**

a) Determine $\frac{d}{dx}(xe^x)$.

$$= e^x + xe^x \quad \checkmark$$

b) Hence evaluate $\int_0^1 \frac{xe^x}{2}\,dx$.

since $\frac{d}{dx}(xe^x) = e^x + xe^x$

✓ Recognise to integrate & and sets up eqn.

then $\int_0^1 xe^x\,dx = \int \frac{d}{dx} xe^x\,dx - \int e^x\,dx.$

✓ use FTC to simplify xe^x

$$\int_0^1 xe^x\,dx = \underbrace{\left[xe^x\right]_0^1 - \left[e^x\right]_0^1}$$

✓ sets up correct antidiffs with limits.

$$= e^1 - 0 - (e^1 - 1)$$

$$= 1$$

$$\therefore \int_0^1 \frac{xe^x}{2}\,dx = \frac{1}{2} \quad \checkmark$$

evaluates correctly

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MATHEMATICS METHODS : UNITS 3 & 4, 2021

Test 2 – (10%)

3.2.1 to 3.2.22 (not 3.2.5), 3.1.1 – 3.1.6, 3.1.9

Time Allowed	First Name	Surname	Marks
25 minutes			25 marks

Circle your Teacher's Name: Mrs Alvaro, Mrs Bestall, Ms Chua, Mr Gibbon, Mrs Greenaway, Mr Luzuk, Mrs Murray, Ms Robinson, Mr Tanday

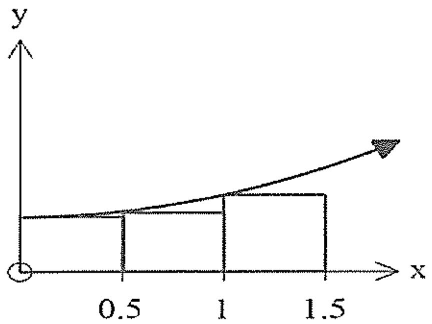
Assessment Conditions: ***(N.B. Sufficient working out must be shown to gain full marks)***

- Calculators: Allowed
- Formula Sheet: Provided
- Notes: Not Allowed

PART B – CALCULATOR ALLOWED

Question 1 **[3,1 — 4 marks]**

The graph of the function *g(x)* is shown below. The table gives the value of the function at the given value of *x*. The rectangles drawn on the graph can be used to underestimate the area under the curve. Other rectangles could be used to overestimate the area.



x	0	0.5	1	1.5
g(x)	9	10	12	15

a) By considering the areas of these rectangles, show how you could estimate $\int_0^{1.5} g(x)\,dx$.

underestimate $= 0.5(9+10+12)$
$= 15.5$ ✓

overestimate $= 0.5(10+12+15)$
$= 18.5$ ✓

Estimate $= \dfrac{15.5+18.5}{2}$

$= 17$ ✓ units^2 (ignore units^2).

b) State whether your estimate above is too large or too small and suggest a modification to the numerical method employed to obtain a more accurate estimate.

Too large, use more rectangles for estimate / make rectangles narrower. ✓ for both correct, allow any reasonable modification.

Question 2 **[1, 2, 2 —5 marks]**

Demographers monitored a particular city's population growth P, in thousands, and found it grew according to the model $P = 22.4e^{kt}$, where t is the time in months from 1st January, 2010.

(a) What was the population of the city on 1st January, 2010?

22400 ✓

By the 1st February, 2010, the population of the city increased by 250 people.

(b) Determine the value of k, rounding your answer to 4 decimal places.

$22.65 = 22.4e^{K \times 1}$ ✓

$K = 0.0111$ ✓ must be correct to 4dp.

(c) Determine the rate of change of the population of the city on 1st January, 2011.

$\frac{dP}{dt} = 0.0111 \times 22.4e^{0.0111 \times 12}$ ✓ sets up correct eqn using $t = 12$.

$\approx 0.0111 \times 25.5912$

≈ 0.284 thousand people / month

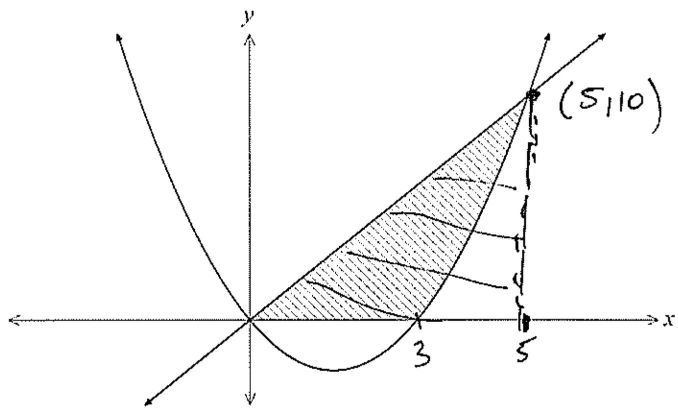
≈ 284 people / month ✓ correct answer and units.

(-1 if correct answer incorrect/no units)

Question 3 **[5 marks]**

The diagram shows a sketch of the curve $y = x(x-3)$ and the line $y = 2x$.

Find the area of the shaded region shown.



Points of Intersection $(0,0)$ & $(5,10)$ ✓ identifies pts int

x-Intercept Parabola. $(3,0)$ ✓ identifies root.

(O.K if points just used as limits for integrals)

$A(\text{Large Tri}) = \frac{1}{2} \times 5 \times 10 = 25$ ✓

$A(\text{under curve}) = \int_3^5 x(x-3)\,dx = \frac{26}{3}$ ✓

$A(\text{Shaded}) = 25 - \frac{26}{3} = 16\frac{1}{3}$ ✓ units^2

Question 4 **[2, 2, 2 — 6 marks]**

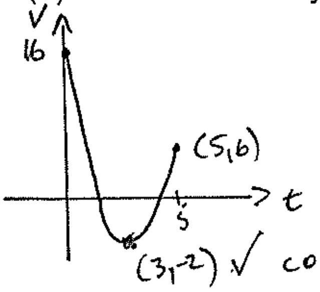
A body moves along a straight line such that the velocity, at time t seconds, is given by v metres per second where $v = 2t^2 - 12t + 16$.
The initial displacement of the body from the origin O is 4 metres.

(a) Find an expression for the displacement of the particle from O at time t.

$$x = \underbrace{\frac{2t^3}{3} - 6t^2 + 16t}_{\checkmark \text{ correct antidiff}} + 4 \checkmark$$

correct c.

(b) When is the body moving the fastest in the first 5 seconds?



√ correct T.P.

$t = 0$ sec ✓

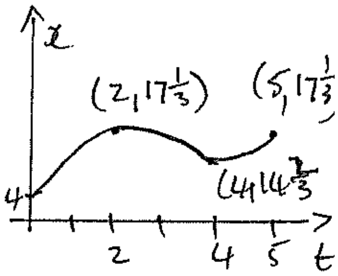
answer only O.K.

(c) The total distance travelled by the particle in the first 5 seconds

$$\int_0^5 |2t^2 - 12t + 16| \, dt \checkmark$$

$$= 18\tfrac{2}{3} \checkmark \text{ m}$$

or



✓

$13\frac{1}{3} + 2\frac{2}{3} + 2\frac{2}{3}$

$= 18\frac{2}{3}$ m

✓

Question 5 **[5 marks]**

Let $k(x) = \int_{-1}^{x} g(t)\, dt$ with $k(4) = 20$ and $\frac{d^2k}{dx^2} = 2x$.
Determine the function $g(x)$.

Since $k(x) = \int_{-1}^{x} g(t)\, dt$

then $k'(x) = g(x)$ ✓ use FTC to form eq^{n}.

$k''(x) = g'(x) = 2x$ ✓ states $k''(x) = 2x$.

$k'(x) = x^2 + c = g(x)$ ✓ recognises $g(x) = x^2 + c$

straight to this O.K.

since $k(x) = \int_{-1}^{x} g(t)\, dt$

then $k(x) = \int_{-1}^{x} t^2 + c\, dt$

$k(4) = \left[\frac{x^3}{3} + cx\right]_{-1}^{4} = 20$ ✓

integrates there $g(t)$ to form eq^{n} = 20 with correct upper & lower limits

$$\frac{64}{3} + 4c - \left(-\frac{1}{3} - c\right) = 20$$

$$c = -\frac{1}{3}$$

$g(x) = x^2 - \frac{1}{3}$ ✓ solves for c and states $g(x)$

(2 slips with x/t usage O.K, more than 2, -1 mark)
please check mine :-)